RECOMP II USER'S PROGRAM NO. 1119

PROGRAM TITLE:	CURVE GENERATION
PROGRAM CLASSIFICATION:	General
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PURPOSE:	To find the linear function of one curve which best approximates a second curve by the least mean squares method. The percent error of approximation is also determined.
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1. INTRODUCTION

It is often desirous to have a medium of correspondence between two sets of data, empirical or otherwise, when there exists a degree of linearity between the functional representations of the data. This program calculates the linear function of one curve which best approximates a second curve by the least mean squares method. One curve X can be used to generate curves Y_i by fitting X over each Y_i in the best linear fashion. The program was principally designed to economize on time and non-linear equipment in analog computer empirical curve simulation.

2. METHOD

2.1 Let curve X and curve Y be functions, empirical or otherwise, of the same variable Z. Choose N points from curve Y (the curve to be generated) at Z_i and N corresponding points from curve X at the same Z_i. Setting $Y_a = A + BX$, the problem now resolves in finding the values of A and B which make the mean square value of $Y - Y_a$ a minimum. The mean square of $Y - Y_a$ =

$$\frac{\sum_{i=1}^{N} (Y_i - Y_{ai})^2}{N} = \frac{\sum_{i=1}^{N} (Y_i)^2}{N} - 2\frac{\sum_{i=1}^{N} Y_i Y_{ai}}{N} + \frac{\sum_{i=1}^{N} (Y_{ai})^2}{N}$$

This reduces to

Equation I:
$$\frac{\sum_{i=1}^{N} (Y_i)^2}{N} - 2A\frac{\sum_{i=1}^{N} Y_i}{N} - 2B\frac{\sum_{i=1}^{N} Y_i X_i}{N}$$
$$+ A^2 + 2AB\frac{\sum_{i=1}^{N} X_i}{N} + B^2\frac{\sum_{i=1}^{N} (X_i)^2}{N} = F(A,B)$$

F(A,B) has a minimum with respect to A at $\frac{\partial(\text{mean square } (Y-Y_a))}{\partial A} = 0$.

Hence, $A = Y_i - BX_i$. Substituting for A in equation I and minimizing we have

$$B = \left(\frac{\sum_{i=1}^{N} Y_i X_i}{N} - \frac{\sum_{i=1}^{N} X_i \sum_{i=1}^{N} Y_i}{N}\right)\left(\frac{\sum_{i=1}^{N} (X_i)^2}{N} - \left(\frac{\sum_{i=1}^{N} X_i}{N}\right)^2\right)^{-1}$$

The percentage error at each point is computed as

$$(P.E.)_i = \frac{100(Y_i - Y_{ai})}{Y_i} \qquad 0 < i \leq N$$

3. RESTRICTIONS

3.1 Components Required

The photoreader and typewriter are required.

3.2 Subroutines utilized.

3.2.1 Floating Point Input AN-007.1

3.2.2 Floating point to fixed point output AN-015.1

3.2.3 Decimal output AN-016.0

3.3 $2 \leq N \leq 10$ where N is the number of sets of points from both curves. It is obvious that N varies with the complexity of the curve being generated. Thus, in generating a straight line from a straight line, only two sets of points need to be chosen along the common variable Z. A more complex curve would require N to be larger for increased accuracy. (Note: Judicious selection of Z_i's will improve approximation; i.e., points should be chosen at slope intersections on curve Y).

3.4 No point may be chosen which has a value of zero.

3.5 If curve X is a straight line with slope zero, 3.3 becomes $2 < N \leq 10$. Failure to meet this condition results in a possible overflow.

4. USAGE

4.1 Computer set-up.

4.1.1 Set sense-switch C down if heading is desired.

4.1.2 All other switches are in normal position.

4.2 Sequence of manual operations.

4.2.1 Load and verify program tape. The program will print out heading, if sense switch C is on, and halt waiting for input data. If the tape does not have L 0651.0 S at the end, press start 1 to achieve the same results.

4.2.2 Set sense switch B down, and press start 1. The typewriter will carriage return and print "N:". Enter, on the typewriter, the number of pairs of points chosen from curves X and Y by typing an integer N, $2 \leq N \leq 10$.

4.2.3 Begin entering data. Enter N values, $2 \leq N \leq 10$, from the Y curve first, followed by the corresponding N values from the X curve. The numbers may be integers, fractions, or mixed numbers.

4.3 Computation

4.3.1 When all the data has been entered, depresss the start button to initiate computation. The values of A and B of the equation Y = A + BX will be typed out. The program will also type the difference between Y and the actual curve generated at each discrete point, along with the percent error at that point. The average percent error is also determined.

4.4 Error codes and restart procedures.

4.4.1 If a correction is to be made in the data, put sense switch D down and press start 2. Enter first, on the typewriter, the number of correct pieces of data preceding the incorrect entry. Then re-enter the correct piece(s) of data. After corrections have been made, put sense switch D up, press error reset and start 2 to begin calculations.

4.4.2 Computation Errors

No errors are anticipated.

4.5 Extent of Storage Requirements.

4.5	Program Locations	0470_8 to 1200_8
		1250_8 to 1447_8
		0001_8 and 0002_8
4.5.2	Subroutine Locations	0010_8 to 0467_8
4.5.3	Data Locations	1200_8 to 1247_8

4.6 Output Format.

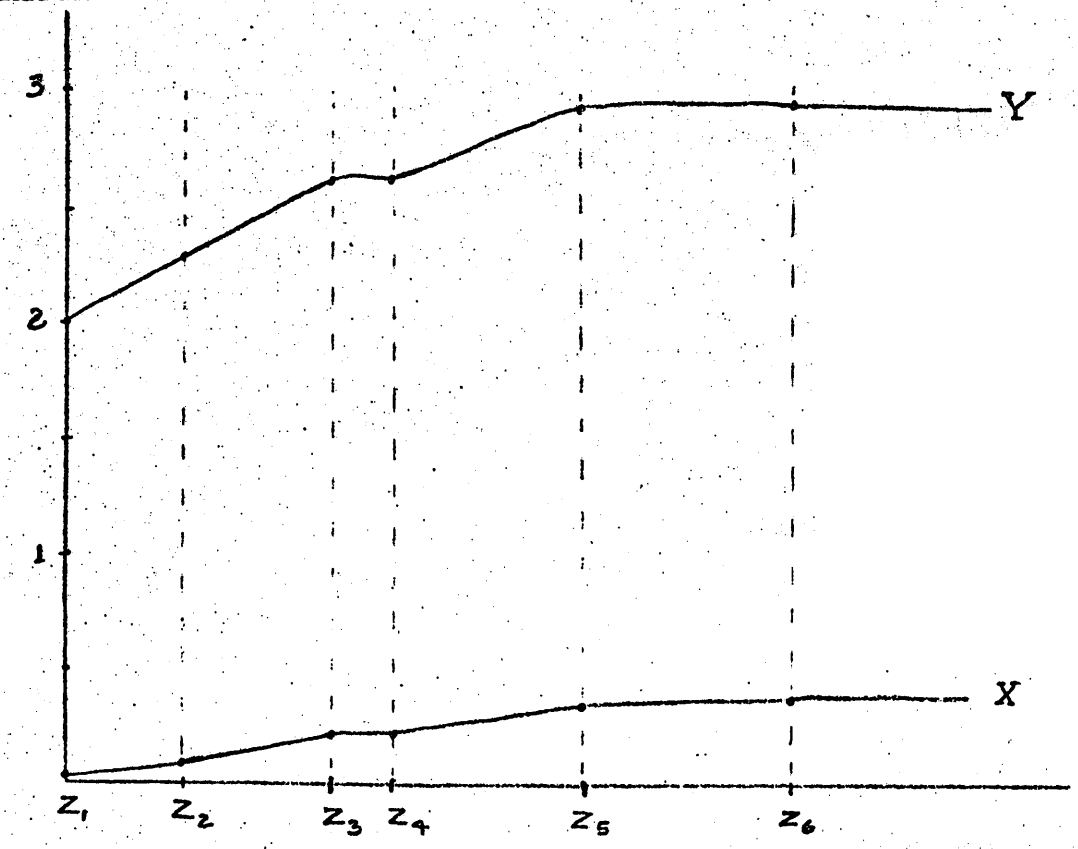
4.6.1 The output format is the same as that shown in the example problem where:

(1) DYi ($1 \leq i \leq 10$) is the difference at point i between the actual curve generated and the Y curve.

(2) P.E. is the percent error at the corresponding point.

(3) E (Average) is the average percent error.

5.0 EXAMPLE



$$Y_R = A + BX$$

CURVE GENERATION

THE PURPOSE OF THIS PROGRAM IS TO GIVE THE BEST APPROXIMATION OF ONE CURVE AS A LINEAR FUNCTION OF ANOTHER WHEN BOTH CURVES ARE FUNCTIONS OF THE SAME VARIABLE. THE PERCENT ERROR OF APPROXIMATION IS ALSO DETERMINED.

```
N:6
2                    .0001
2.3                  .1
2.6                  .2
2.6                  .2
2.9                  .3
2.9                  .3

A:  1.99980241334
B:  3.00080494417
DY1:-      .000102        P.E.-      .005125
DY2:       .000117        P.E.       .005091
DY3:       .000037        P.E.       .001408
DY4:       .000037        P.E.       .001408
DY5:-      .000044        P.E.-      .001514
DY6:-      .000044        P.E.-      .001514
E(AVERAGE):-         .0000
```

6.0 CODING INFORMATION

6.1 Constants and their locations (Octal).

6.1.1 Fixed Point

Constant		Location
1060 at B18	In	0473

6.1.2 Floating Point

Constant	Location
100.0	0470
2.0	0476
0.0	1260
1.0	1262

6.2 Timing Estimate.

6.2.1 (16 + 10N) seconds, when N is the number of points chosen from the common variable of the X and Y curves.